Pre-class Warm-up !!

What does Fubini's theorem say?

a. The value of the integral is the volume under the graph.

b. Continuous functions are integrable.

c. The integral is the limit of Riemann sums.

d. We can integrate in either order and get the same answer.

e. The derivative of the integral is the function we first thought of.

- Exam 1 is on Tuesday in your discussion section.
- It is 50 minutes long. No calculators. You can have a single sheet of notes. There are 5 questions, some with two parts. Show your work so we can see you know how to do the question.
- It is on everything we have done through 2.6.
- There are past exams on the Canvas site in the very first module. Practice doing them.
- The exam questions are similar to questions from the book listed for HW. Practice those questions so you can do them fluently, without thinking.
- There is a quiz on Thursday next week.

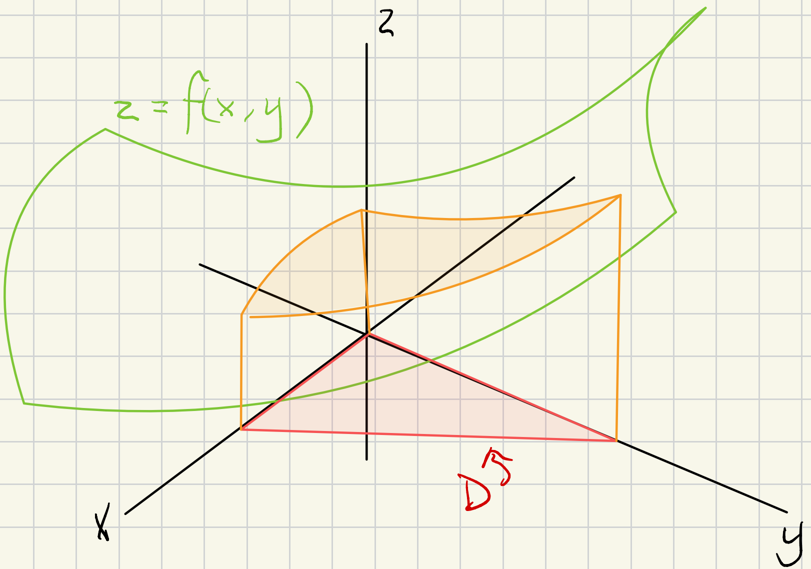
5.3 Double integrals over more general regions

In 5.1 and 5.2 we did integrals over rectangles. Now we do integrals over other regions, like triangles, circles, regions bounded by curves

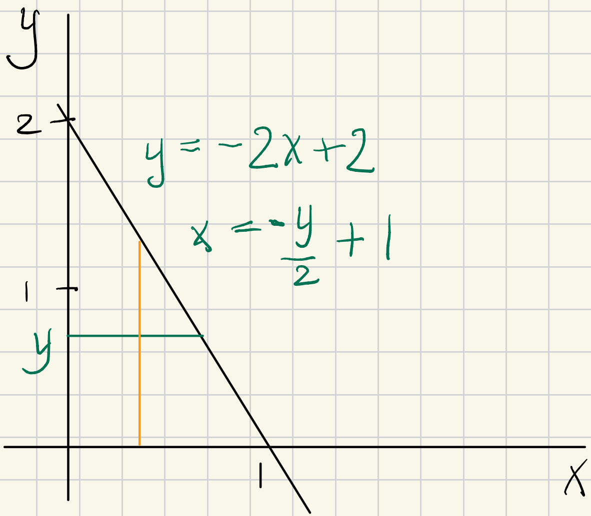
We learn:

- How to set up the integral as a double integral so that it goes over some funny shape.
- How they justify this procedure in the book
- Vocabulary: x-simple, y-simple
- elementary, simple region

$$\iint_D x^2 y \, dA$$ where D is the triangle with vertices $(0,0), (1,0), (0,2)$



$$\iint_D x^2 y \, dA \quad \text{where } D \text{ is the triangle with vertices } (0,0), (1,0), (0,2)$$



For each value of y we integrate w.r.t. x

Integrate first w.r.t y

$$\iint_D x^2 y \, dA = \int_0^2 \int_0^{-y/2+1} x^2 y \, dx \, dy$$

$$= \int_0^2 \left[\frac{x^3 y}{3}\right]_0^{-\frac{y}{2}+1} dy = \int_0^2 \frac{\left(-y/2+1\right)^3 y}{3} \, dy$$

$$= \frac{1}{3}\int_0^2 \left(-\frac{y^4}{8} + \frac{3y^3}{4} - \frac{3y^2}{2} + y\right) dy$$

$$= \frac{1}{3}\left[-\frac{y^5}{40} + \frac{3y^4}{16} - \frac{y^3}{2} + \frac{y^2}{2}\right]_0^2 = \frac{1}{3}\left(-\frac{32}{40} + 3 - 4 + 2\right)$$

$$= \frac{1}{15}$$

Or

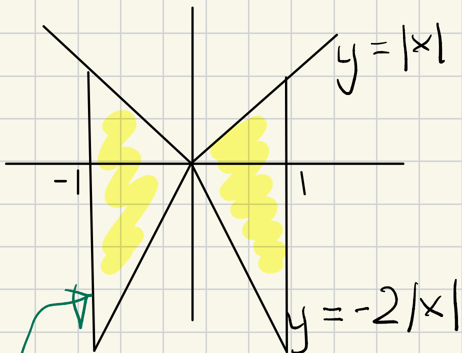
$$\int_0^1 \int_0^{-2x+2} x^2 y \, dy \, dx = \int_0^1 \left[\frac{x^2 y^2}{2}\right]_0^{-2x+2} dx$$

$$= \int_0^1 \frac{x^2(-2x+2)^2}{2} dx = \int_0^1 \frac{4x^4 - 8x^3 + 4x^2}{2} dx$$

$$= \left[\frac{2x^5}{5} - x^4 + \frac{2x^3}{3}\right]_0^1 = \frac{2}{5} - 1 + \frac{2}{3} = \frac{1}{15}$$

Exercise 4b page 288: sketch the region of integration: $$\int_{-1}^{1}\int_{-2|x|}^{|x|} e^{x+y}\,dy\,dx$$

Solution: The integration w.r.t. y starts at $y=-2|x|$ and ends at $y=|x|$

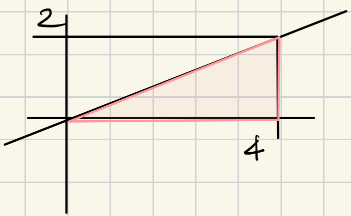


region of integration.

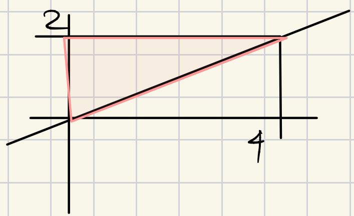
Match the region of integration to the shape shown:

$$\int_0^2 \int_{2y}^4 f\,dx\,dy$$

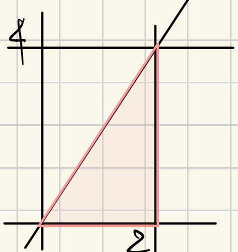
a. ✓



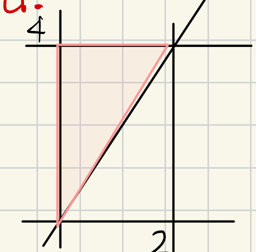
b.



c.



d.



x goes between $x=2y$ $(y=\frac{1}{2}x)$ and $x=4$, y goes between 0 and 2

b.

What about: $\int_0^4 \int_{\frac{x}{2}}^2 f\,dy\,dx$

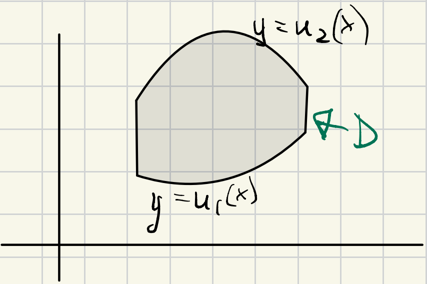
$\int_0^2 \int_0^{2y} f\,dx\,dy$ b.

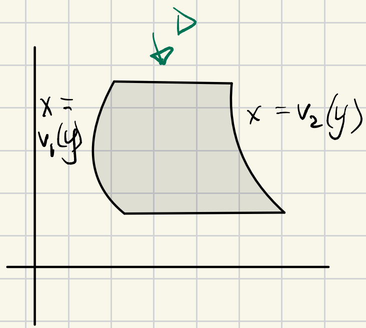
$\int_0^4 \int_0^{\frac{x}{2}} f\,dy\,dx$? a.

The theory behind it

They define the integral on regions of the plane between two graphs
$y = u_1(x)$ and $y = u_2(x)$ or

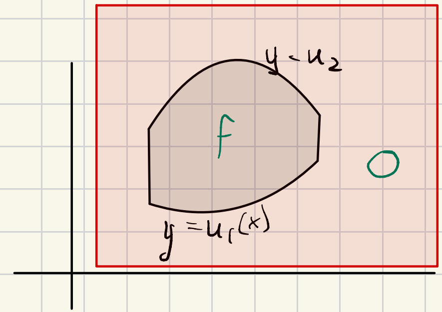
$x = v_1(y)$ and $x = v_2(y)$



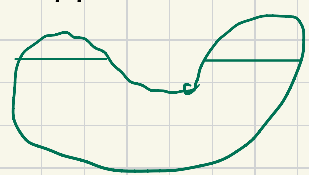


In 5.2 they had a theorem that a function continuous except on graphs of functions is integrable.

They enclose the region D between two graphs inside a bigger rectangle R, and integrate a function that is f on D and zero on R outside D.



This region is called y-simple, the other is called x-simple. Simple means either, and appears to be the same as elementary.



y simple
not x simple.